components. Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of product 1, 1 unit of frame parts and 2 units of electrical components are required. For each unit of product 2, 3 units of frame parts and 2 units of electrical components are required. The company has 200 units of frame parts and 300 units of electrical components. Each unit of product 1 gives a profit of $1, and each unit of product 2, up to 60 units, gives a profit of $2. Any excess over 60 units of product 2 brings no profit, so such an excess has been ruled out.
(a) Formulate a linear programming model for this problem.
D,I (b) Use the graphical method to solve this model. What is the resulting total profit?

3.1-9. The Primo Insurance Company is introducing two new product lines: special risk insurance and mortgages. The expected profit is $5 per unit on special risk insurance and $2 per unit on mortgages.

Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

	Work-Hours per Unit		
Department	**Special Risk**	**Mortgage**	**Work-Hours Available**
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

(a) Formulate a linear programming model for this problem.
D,I (b) Use the graphical method to solve this model.
(c) Verify the exact value of your optimal solution from part (*b*) by solving algebraically for the simultaneous solution of the relevant two equations.

3.1-10. Weenies and Buns is a food processing plant which manufactures hot dogs and hot dog buns. They grind their own flour for the hot dog buns at a maximum rate of 200 pounds per week. Each hot dog bun requires 0.1 pound of flour. They currently have a contract with Pigland, Inc., which specifies that a delivery of 800 pounds of pork product is delivered every Monday. Each hot dog requires $\frac{1}{4}$ pound of pork product. All the other ingredients in the hot dogs and hot dog buns are in plentiful supply. Finally, the labor force at Weenies and Buns consists of 5 employees working full time (40 hours per week each). Each hot dog requires 3 minutes of labor, and each hot dog bun requires 2 minutes of labor. Each hot dog yields a profit of $0.80, and each bun yields a profit of $0.30.

Weenies and Buns would like to know how many hot dogs and how many hot dog buns they should produce each week so as to achieve the highest possible profit.
(a) Formulate a linear programming model for this problem.
D,I (b) Use the graphical method to solve this model.

3.1-11.* The Omega Manufacturing Company has discontinued the production of a certain unprofitable product line. This act created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; call them products 1, 2, and 3. The available capacity on the machines that might limit output is summarized in the following table:

Machine Type	Available Time (Machine Hours per Week)
Milling machine	500
Lathe	350
Grinder	150

The number of machine hours required for each unit of the respective products is

Productivity coefficient (in machine hours per unit)

Machine Type	Product 1	Product 2	Product 3
Milling machine	9	3	5
Lathe	5	4	0
Grinder	3	0	2

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and that the sales potential for product 3 is 20 units per week. The unit profit would be $50, $20, and $25, respectively, on products 1, 2, and 3. The objective is to determine how much of each product Omega should produce to maximize profit.
(a) Formulate a linear programming model for this problem.
C (b) Use a computer to solve this model by the simplex method.

D **3.1-12.** Consider the following problem, where the value of c_1 has not yet been ascertained.

Maximize $Z = c_1x_1 + x_2$,

subject to

$$x_1 + x_2 \le 6$$
$$x_1 + 2x_2 \le 10$$

and

$$x_1 \ge 0, \quad x_2 \ge 0.$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of $c_1 (-\infty < c_1 < \infty)$.

D **3.1-13.** Consider the following problem, where the value of k has not yet been ascertained.

Maximize $Z = x_1 + 2x_2$,

Excel Add-ins:

Premium Solver for Education
Solver Table

"Ch. 12—Nonlinear Programming" Files for Solving the Examples:

Excel Files
LINGO/LINDO File
MPL/CPLEX/CONOPT/LGO File

Glossary for Chapter 12

See Appendix 1 for documentation of the software.

PROBLEMS

The symbols to the left of some of the problems (or their parts) have the following meaning:

D: The corresponding demonstration example just listed in Learning Aids may be helpful.
I: We suggest that you use the corresponding interactive routine just listed (the printout records your work).
C: Use the computer with any of the software options available to you (or as instructed by your instructor) to solve the problem.

An asterisk on the problem number indicates that at least a partial answer is given in the back of the book.

12.1-1. Read the referenced article that fully describes the OR study summarized in the application vignette presented in Sec. 12.1. Briefly describe how nonlinear programming was applied in this study. Then list the various financial and nonfinancial benefits that resulted from this study.

12.1-2. Consider the *product mix* problem described in Prob. 3.1-11. Suppose that this manufacturing firm actually encounters *price elasticity* in selling the three products, so that the profits would be different from those stated in Chap. 3. In particular, suppose that the unit costs for producing products 1, 2, and 3 are \$25, \$10, and \$15, respectively, and that the prices required (in dollars) in order to be able to sell x_1, x_2, and x_3 units are $(35 + 100x_1^{-\frac{1}{3}})$, $(15 + 40x_2^{-\frac{1}{4}})$, and $(20 + 50x_3^{-\frac{1}{2}})$, respectively.

Formulate a nonlinear programming model for the problem of determining how many units of each product the firm should produce to maximize profit.

12.1-3. For the P & T Co. problem described in Sec. 8.1, suppose that there is a 10 percent discount in the shipping cost for all truckloads *beyond* the first 40 for each combination of cannery and warehouse. Draw figures like Figs. 12.3 and 12.4, showing the marginal cost and total cost for shipments of truckloads of peas from cannery 1 to warehouse 1. Then describe the overall nonlinear programming model for this problem.

12.1-4. A stockbroker, Richard Smith, has just received a call from his most important client, Ann Hardy. Ann has \$50,000 to invest, and wants to use it to purchase two stocks. Stock 1 is a solid blue-chip security with a respectable growth potential and little risk involved. Stock 2 is much more speculative. It is being touted in two investment newsletters as having outstanding growth potential, but also is considered very risky. Ann would like a large return on her investment, but also has considerable aversion to risk. Therefore, she has instructed Richard to analyze what mix of investments in the two stocks would be appropriate for her.

Ann is used to talking in units of thousands of dollars and 1,000-share blocks of stocks. Using these units, the price per block is 20 for stock 1 and 30 for stock 2. After doing some research, Richard has made the following estimates. The expected return per block is 5 for stock 1 and 10 for stock 2. The variance of the return on each block is 4 for stock 1 and 100 for stock 2. The covariance of the return on one block each of the two stocks is 5.

Without yet assigning a specific numerical value to the minimum acceptable expected return, formulate a nonlinear programming model for this problem.

12.2-1. Reconsider Prob. 12.1-2. Verify that this problem is a convex programming problem.

12.2-2. Reconsider Prob. 12.1-4. Show that the model formulated is a convex programming problem by using the test in Appendix 2 to show that the objective function being minimized is convex.

12.2-3. Consider the variation of the Wyndor Glass Co. example represented in Fig. 12.5, where the second and third functional constraints of the original problem (see Sec. 3.1) have been replaced by $9x_1^2 + 5x_2^2 \leq 216$. Demonstrate that $(x_1, x_2) = (2, 6)$ with $Z = 36$ is indeed optimal by showing that the objective function line $36 = 3x_1 + 5x_2$ is *tangent* to this constraint boundary at (2, 6). (*Hint:* Express x_2 in terms of x_1 on this boundary, and then differentiate this expression with respect to x_1 to find the slope of the boundary.)